

$$P_{\text{rest}} = \int dx \psi^* \left[ \frac{1}{2m} \left( \frac{\hbar}{i} \frac{\partial}{\partial x} \right)^2 \right] \psi + \psi^* V \psi$$

$$+ \frac{\hbar^2}{2m} \int dx \psi \frac{\partial^2 \psi^*}{\partial x^2} - \psi \frac{\partial^2 \psi}{\partial x^2}$$

$$= -\frac{\hbar^2}{2m} \left[ \int \psi^* \frac{\partial}{\partial x} \frac{\partial \psi}{\partial x} dx - \int \psi \frac{\partial}{\partial x} \frac{\partial \psi^*}{\partial x} dx \right]$$

$$= -\frac{\hbar^2}{2m} \left[ - \int \psi^* \frac{\partial \psi}{\partial x} dx + 0 + \int \psi \frac{\partial \psi^*}{\partial x} dx - 0 \right]$$

$$= -\frac{\hbar^2}{2m} \left( \int \psi^* \psi dx - \psi^* \psi - \int \psi \psi^* dx + \psi \psi^* \right)$$

$$\Rightarrow \left[ \frac{d\langle p \rangle}{dt} = - \left\langle \frac{\partial V}{\partial x} \right\rangle \right] \quad \text{Proved.}$$

Problem 1.8. Suppose you add a constant  $V_0$  to the point potential energy. (by constant: independent of  $x$  or  $t$ ) In classical mechanics this doesn't change anything. but what about quantum mechanics? Show that the wave function picks up a time dependent phase factor:  $\exp(-iV_0 t/\hbar)$ . What effect does this have on expectation values of dynamic variables?

Solution: Use Schrödinger's eq<sup>n</sup> for  $\psi = \psi e^{-iV_0 t/\hbar}$

and use  $V \Rightarrow V_0 + V$

expectation value remains same as its a phase factor and its complex conjugate cancel s.