Problem 1.9:

A particle of mass m is in the state

$$\Psi(x,t) = Ae^{-a[(mx^2/\hbar)+it]}$$ where

A and a are positive real constants.

(a) Find A

(b) For what pot. energy function V(x) does Ψ satisfy the Schrodinger's eq^n ?

(c) Calculate the expectation values of x, x^2, p and p^2.

(d) Find σ_x and σ_p. Is their product consistent with the Uncertainty principle?

Solution: $\int \Psi^* \Psi \, dx = A^2 \int e^{-2a\, mx^2/\hbar} dx = 1$

or $A^2 \int_{-\infty}^{\infty} e^{-2am/\hbar\, x^2} dx = 1$ or $A^2 \sqrt{\frac{\pi\hbar}{2am}} = 1$

or $A = \left(\frac{2ma}{\pi\hbar}\right)^{\frac{1}{4}}$.

(b) $i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi$

or $i\hbar(-ai)\Psi = -\hbar^2/2m \frac{\partial}{\partial x}\left[-\frac{2ma}{\hbar} x\Psi\right] + V(x)\Psi$

or $a\hbar\Psi = -\frac{\hbar^2}{2m} \frac{2ma}{\hbar}\left[\Psi + x\left(-\frac{2ma}{\hbar} x\Psi\right)\right] + V(x)\Psi$

or $a\hbar\Psi = -\hbar a\left[\Psi - \frac{2ma}{\hbar} x^2\Psi\right] + V(x)\Psi$

or $a\hbar\Psi = a\hbar\left[\frac{2ma}{\hbar} x^2 - 1\right]\Psi + V(x)\Psi$

or $V(x) = 2a\hbar\left(1 - \frac{ma}{\hbar} x^2\right)$