Problem 1.3. $f(x) = Ae^{-\lambda(x-a)^2}$, is a Gaussian dist

A, a, λ are real positive constants.

(a) Use eqn 1.16 to determine A

(b) Find $\langle x \rangle$, $\langle x^2 \rangle$ and σ

(c) sketch the graph of $f(x)$

(a) $1 = \int_{-\infty}^{\infty} f(x)\,dx$, $\int_{-\infty}^{\infty} A e^{-\lambda(x-a)^2} dx = 1$

Since $\int_0^{\infty} x^{2s} e^{-ax^2} dx = \dfrac{(2s-1)!!}{2^{s+1} a^s} \sqrt{\pi/a}$ (Arfken)

for $s = 0$ $\int_0^{\infty} e^{-ax^2} dx = \frac{1}{2}\sqrt{\pi/a}$, $\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\pi/a}$

In our integration, put, $x - a = y$, $dx = dy$

$$\int_{-\infty}^{\infty} Ae^{-\lambda y^2} dy = A\sqrt{\pi/\lambda} = 1$$

$$\therefore \boxed{A = \sqrt{\lambda/\pi}}$$

$$\langle x \rangle = \int_{-\infty}^{\infty} x \cdot Ae^{-\lambda(x-a)^2} dx$$

Since $\int_0^{\infty} x^{2s+1} \exp(-ax^2)\,dx = \dfrac{s!}{2a^{s+1}}$

for $s = 0$

$$\int_{0}^{\infty} x \exp(-ax^2)\,dx = \frac{1}{2a}.$$

our integration, put $x - a = y$, $dx = dy$

$$= A\int_{-\infty}^{\infty} (y+a)\, e^{-\lambda y^2} dy$$

$$= A\left[\int_{-\infty}^{\infty} y e^{-\lambda y^2} dy + 2a\int_0^{\infty} e^{-\lambda y^2} dy\right] = a$$