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$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 \cdot A e^{-\lambda(x-a)^2} dx$$

$$= A \int_{-\infty}^{\infty} (y+a)^2 \cdot e^{-\lambda y^2} dy$$

$$= A \left[ \int_{-\infty}^{\infty} y^2 e^{-\lambda y^2} dy + \int_{-\infty}^{\infty} 2ay\, e^{-\lambda y^2} dy + \int_{-\infty}^{\infty} a^2 e^{-\lambda y^2} dy \right]$$

$\to 0$

for $\underline{\text{integral 1}}$: $\int_0^{\infty} x^{2s} e^{-ax^2} dx = \frac{(2s-1)!!}{2^{s+1} a^s} \sqrt{\frac{\pi}{a}}$

for $\underline{s=1}$ $\int_0^{\infty} x^2 e^{-ax^2} dx = \frac{1}{4a} \sqrt{\frac{\pi}{a}}$

integral $1 = 2A \cdot \frac{1}{4\cancel{\lambda}} \sqrt{\pi/\lambda}$, $A = \sqrt{\frac{\lambda}{\pi}}$

$= \frac{1}{2\lambda}$

integral $2 = A \cdot a^2 \cdot \frac{1}{A} = a^2$

$\Rightarrow \langle x^2 \rangle = a^2 + \frac{1}{2\lambda}$

$\langle x \rangle^2 = a^2 \Rightarrow \sigma^2 = \frac{1}{2\lambda}$, or $\sigma = \frac{1}{\sqrt{2\lambda}}$

Ⓒ Gaussian Dist. $\int_{-\infty}^{\infty} A \cdot \exp(-\lambda(x-a)^2) dx$

mean $\langle x \rangle = a$, $\sigma = \frac{1}{\sqrt{2\lambda}}$ or $\lambda = \frac{1}{2\sigma^2}$

$A = \sqrt{\cancel{\lambda}/\cancel{\pi}}$, $A^2 = \cancel{\lambda}/\cancel{\pi} = \frac{1}{2\pi\sigma^2}$ or $A = \frac{1}{\sigma\sqrt{2\pi}}$