$$= \int_0^a x \frac{A^2}{a^2} x^2 dx + \int_a^b x \frac{A^2}{(b-a)^2} (b-x)^2 dx$$

$$= \left(\frac{A}{a}\right)^2 \frac{a^4}{4} + I = \frac{A^2 a^2}{4} + I.$$

$$I = \frac{A^2}{(b-a)^2} \int_a^b (b^2 x + x^3 - 2bx^2)\, dx$$

$$= \frac{A^2}{(b-a)^2} \left[b^2 \left(\frac{b^2 - a^2}{2}\right) + \frac{b^4 - a^4}{4} - 2b \left(\frac{b^3 - a^3}{3}\right) \right]$$

$$= A^2/(b-a)^2 \left[\frac{b^4}{2} - \frac{a^2 b^2}{2} + \frac{b^4}{4} - \frac{a^4}{4} - \frac{2b^4}{3} + \frac{2ba^3}{3} \right]$$

$$= A^2/(b-a)^2 \left[b^4\left(\frac{1}{2} + \frac{1}{4} - \frac{2}{3}\right) + \frac{a^4}{4} - \frac{a^2 b^2}{2} + \frac{2ba^3}{3} \right]$$

$$= \frac{A^2}{(b-a)^2} \left[\frac{b^4}{12} + \frac{a^4}{4} - \frac{a^2 b^2}{2} + \frac{2ba^3}{3} \right].$$

Problem 1.5: Consider the wave function

$\Psi(x,t) = Ae^{-\lambda|x|} e^{-i\omega t}$ where A, λ and ω are positive real constants.

(a) Normalize Ψ. (b) Determine the expectation value of x and x^2.

(c) Find standard deviation of x. Sketch $|\Psi|^2$ as f^n of x and mark points $\langle x\rangle + \sigma$ and $\langle x\rangle - \sigma$ to illustrate the sense in which σ represents the spread in x. What's the probability that the particle would be found in this range.