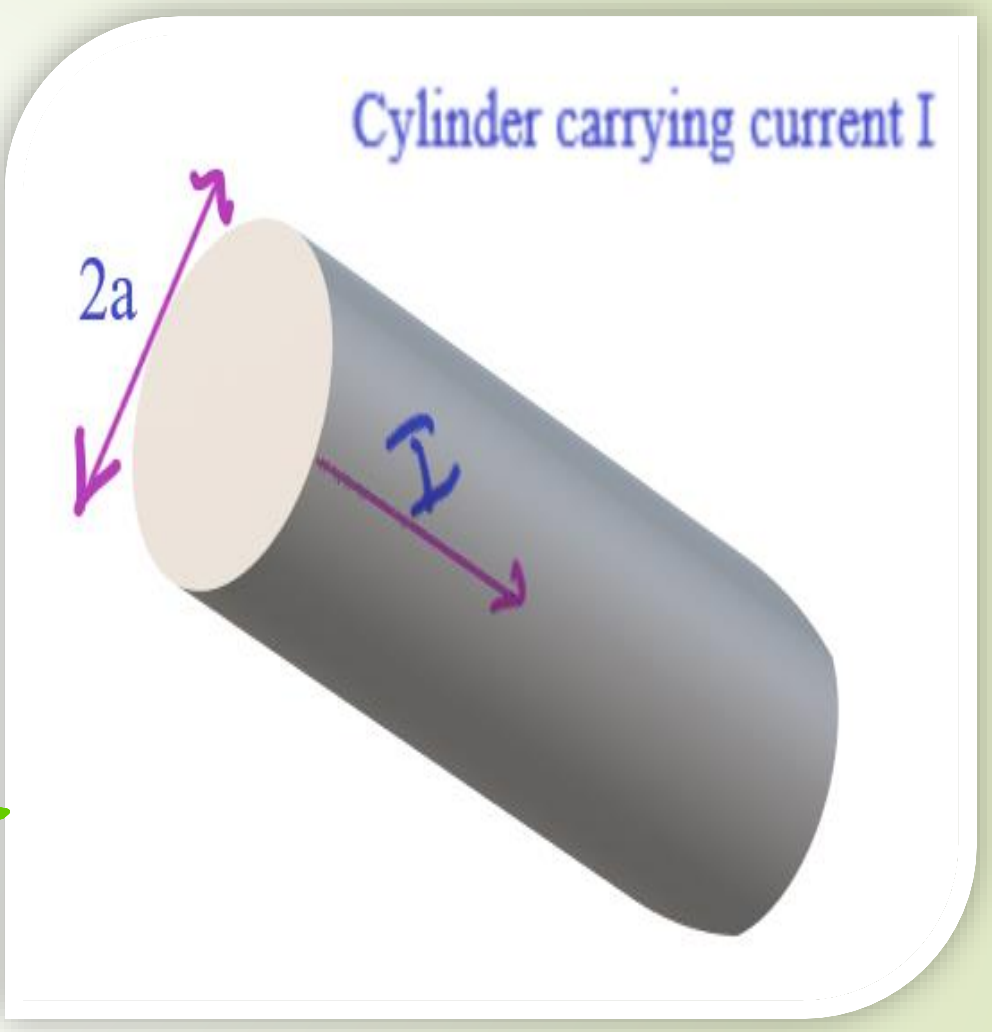
Griffith problem 5.13

- **Problems in Electrodynamics**

5.13 A steady current I flows down a long cylindrical wire of radius a.

Find the magnetic field both outside and inside the wire, if

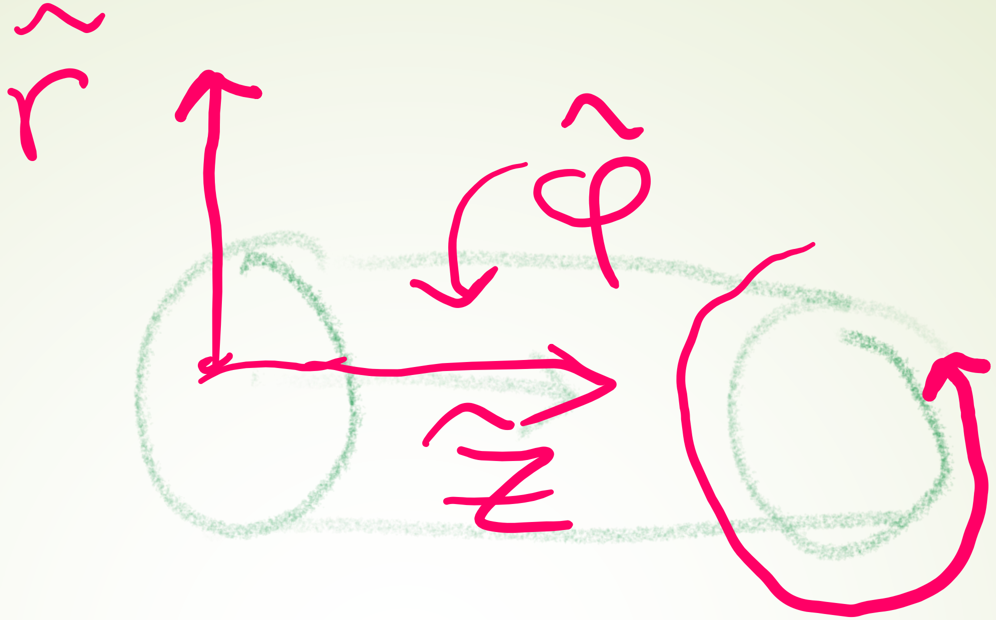


a) The current is uniformly distributed over the outside surface.

b) The current is distributed in such a way that J is proportional to s: the distance from the axis.

(a) Lets apply Ampere's Law,

* Direction: From symmetry and Biot-Savart law its clear that the field is directed around the circumference: $\hat{\varphi}\{\hat{z}\times\hat{r}\}$



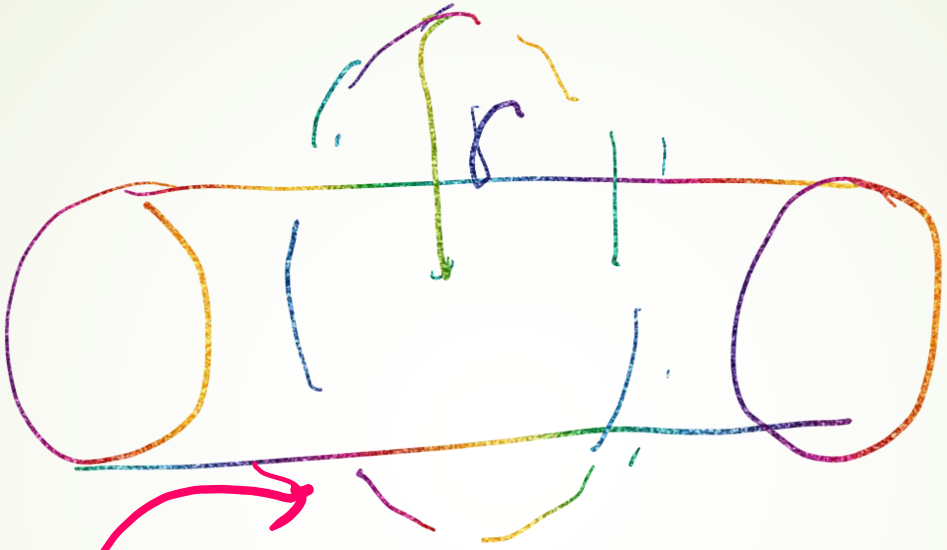
Biot-Savart law: $\frac{\mu_0}{4\pi}\int \frac{\vec{K}\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,da'$

$\vec{K}$ along $\hat{z}$ and $\hat{\mathscr{r}}$ along $\hat{r}$.

* From symmetry magnitude of field, B is uniform on a circle around the cylinder (concentric with its axis)

Now its easy to apply Ampere's Law.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$



Amperian loop

LHS: $B \cdot 2\pi r$

RHS: $\mu_0 I$ if $r > a$

and 0 if $r < a$

$$\Rightarrow B = \begin{cases} \dfrac{\mu_0}{2\pi}\dfrac{I}{a}, & r > a \\ 0, & r < a \end{cases}$$

(5) We first need to find J, the current density.

$\vec{J} = \frac{d\vec{I}}{da_\perp}$. Both $\vec{J}$ and $\vec{I}$ are alon -g $\hat{z}$.

$da_\perp = r\,dr\cdot d\varphi = s\,ds\,d\varphi$; $J = js$, j is const.

$$\int \hat{j}\, s \cdot s\, ds\, d\varphi = I \Rightarrow j\, \frac{s^3}{3}\, 2\pi \Big|_0^a = I$$

or $$j = \frac{3I}{2\pi a^3} \Rightarrow J = j s = \frac{3}{2\pi}\, \frac{I}{a^3}\, s.$$

$$I_{enclosed} = \int J\, da = \frac{3}{2\pi} \int \frac{I}{a^3} \cdot s \cdot s\, ds\, d\varphi$$

$$= I \text{ if } s > a$$

and $$\frac{I}{a^3} s^3 \text{ if } s < a; \quad \frac{\vec{B}}{B} = \hat{\varphi}$$

Now we can apply Ampere's Law.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{enclosed}.$$

$$B \cdot 2\pi s = \mu_0 I_{enc}$$

for $s > a$; $\vec{B} = \dfrac{\mu_0 I}{2\pi s} \hat{\varphi}$

for $s < a$; $\vec{B} = \dfrac{\mu_0 I}{2\pi s} \left(\dfrac{s^3}{a^3}\right) \hat{\varphi}$

$$= \frac{\mu_0 I}{2\pi a^3} s^2 \hat{\varphi}$$